DOUBLE PENDULUM

By William Gurstelle

CHAOS ON A STICK

Devices that demonstrate true chaotic behavior (in a strict mathematical sense) are rare. Even rarer are chaotic devices that are easy enough for the typical maker to build at home and are interesting and beautiful. But one device nicely fits the bill: the double pendulum.

A double pendulum consists of a bar swinging from a pivot, with a second pendulum attached to the first bar's end. While the double pendulum is a simple physical system, you'd be hard pressed to find another device this simple that exhibits so wide a range of behavior. Give it a little push and the motion is fairly predictable. But give it a bigger push — bingo, welcome to chaos!

The double pendulum described here was designed with several options for demonstrating a variety of chaotic motions. With the right mounting, it's an interesting if not downright charming display that fits well into a number of settings, including classrooms, laboratories, and homes.

Photograph by Sam Murphy

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Swing Thing

The Simple, Made Complex

What exactly is chaos? It means different things in different contexts. In common discourse, it means a confused, disordered state of affairs.

To the mathematician or physicist, chaos does not mean arbitrary or random motions and systems, nor does it mean that the outputs of a system are unrelated to its inputs. It does mean that the behavior of a system is, in a practical sense, unpredictable because small differences in initial conditions result in huge differences in subsequent actions. In a chaotic system — like the proverbial Amazon butterfly whose wing-fluttering can affect the weather in Europe — cause and effect are related but the complexity of the system makes accurate predictions impossible.

James Yorke of the University of Maryland is the mathematician who first introduced the term *chaos theory*. "The motion of a double pendulum," he says, "gets pretty complicated. But that's what chaos is."

"[The pendulum is] predictable in the short run but not in the long run," said Yorke in a recent *Washington Post* interview. "Chaos is about lack of predictability. Obviously, the spin of the pendulum is determined by physical laws, but it's very hard to predict because very small changes in the spin cause very big changes in the output."

Simple: Single Pendulum

Chaotic: Double Pendulum

Simple or Chaotic: Rott's Pendulum

Simple: Single Pendulum — Galileo Galilei became interested in pendulums as a university student in Pisa in the 1580s, when he observed a lamp swinging to and fro in a cathedral. His experiments led to his discovery in 1602 that the amount of time it takes for a simple pendulum to swing back and forth is dependent on the length of the pendulum, and not dependent on the weight of the bob or the size of the swing. A half-century later the scientist Christiaan Huygens made use of this feature, known as *isochronism*, by inventing the pendulum clock, which is still in use today.

Chaotic: Double Pendulum — The double pendulum doesn't exhibit isochronism. Though its construction is very simple, its motion is chaotic and impossible to predict, because very small changes in friction, initial drop height, temperature, and other variables have a large effect on its behavior over time. This sensitivity to initial conditions makes the double pendulum interesting to watch, as its pattern of movement is always changing.

Simple or Chaotic: Rott's Pendulum — In this configuration, first analyzed in 1970 by Swiss physicist Nikolaus Rott, a right-angled main pendulum joins a smaller side pendulum, and their 2 pivots are aligned horizontally at rest. When given a gentle push, the 2 arms will move in resonance with one another — but only if the ratio between their fundamental frequencies is 1:2.

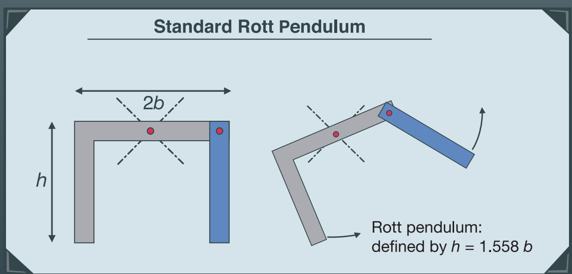
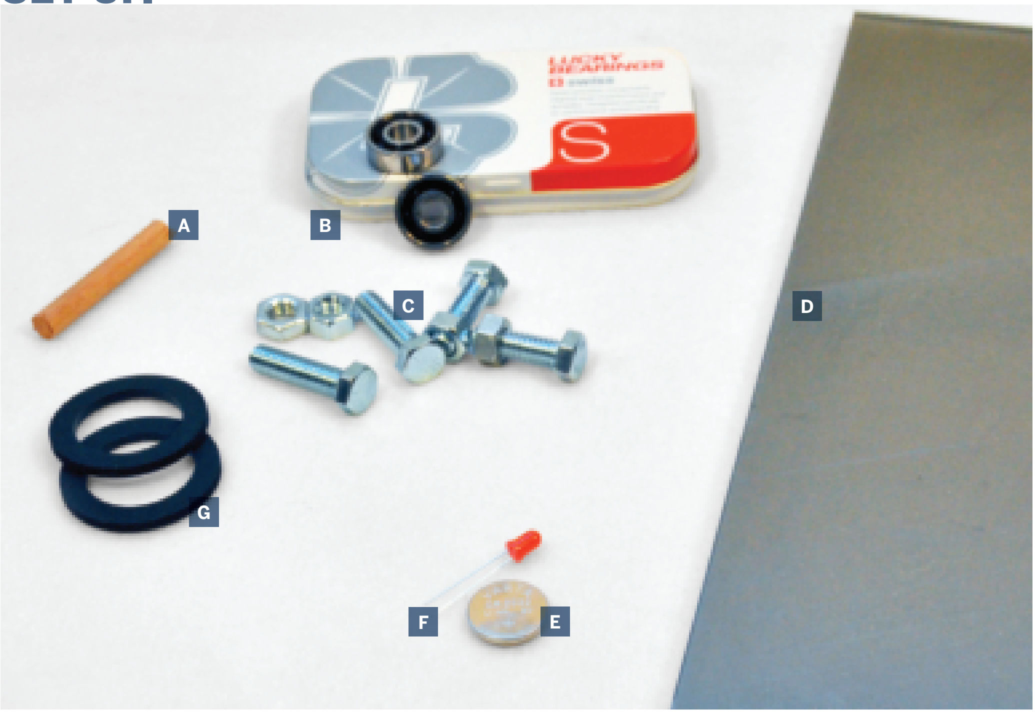


Illustration by Tim Lillis

SET UP.



MATERIALS

The pendulum itself may be fabricated from any number of common materials including plastic, aluminum, and wood; I used polycarbonate plastic.

The exact size of the pendulum may vary according to the wishes of the maker. I ordered my materials from McMaster-Carr (mcmaster.com) because they were easily available and not too expensive; you can also get parts from a local supplier, a surplus dealer, or another mail order supply firm.

[A] 8mm steel rod, 3" length for the main pivot on which the pendulums rotate. 5/16" wood dowel is an acceptable substitute, although you'll need to slightly build it out with tape or foil so it fits the bearing snugly.

[B] Skateboard bearings (7) High-quality bearings that reduce friction to an absolute minimum are the key to getting the best performance from your pendulum.

Skateboard bearings are relatively inexpensive high-performance bearings, sold in skateboard stores in quantities of 8, generally packaged in small metal cans resembling mint tins.

The quality of the bearing is determined by its ABEC designation: ABEC-3 bearings are good, ABEC-5 are better, and ABEC-7 or "Swiss" bearings are great. Buy the best you can afford.

[C] 5/16" bolts, 1" long, with nuts (2) for attaching pendulum bearings

[D] Sheet of polycarbonate plastic 12"×12"×1/4" thick

[E] 3V coin cell battery (optional) to power the LED

[F] LED (optional) for time-lapse photography of pendulum in action

[G] Rubber washers, 1" ID (2)

[NOT SHOWN]

Nylon washers or plastic spacers

8mm shaft collar (optional)

TOOLS

Jigsaw, band saw, or table saw

Rotary tool or file

Rubber mallet

Epoxy glue

Sandpaper

Measuring tape

Electric drill with 22mm or 13/16" spade drill bits

Hacksaw if you're using steel rod

Photography by Ed Troxell

MAKE IT.

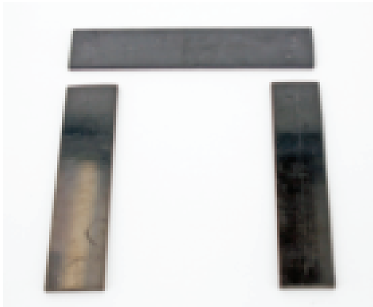
BUILD YOUR DOUBLE PENDULUM

START **Time: 1 Day Complexity: Easy**

1. CUT THE PENDULUM PARTS

Cut the ¼" polycarbonate sheet into 3 pieces as shown in the Part Layout diagram (download at makezine.com/22/doublependulum): one long pendulum piece 2"×12" and 2 short pendulum pieces 2"×8⅝".

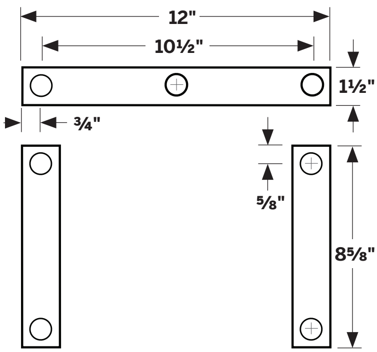
I cut these sizes so the pendulum can be used in a variety of configurations; you can use the long piece and one short piece to make a standard double pendulum, and add the second short piece to make a Rott's pendulum or a triple pendulum.



2. MEASURE AND DRILL BEARING HOLES

The high-performance skateboard bearings measure 22mm wide by 7mm deep, so you must drill a total of seven 22mm-diameter holes in the pendulum parts as shown on the Part Layout diagram, using a spade bit.

If you don't have metric drill bits, you can use a 13⁄16" spade bit and later enlarge the holes slightly with a file or rotary tool.



3. FIT THE BEARINGS IN THE HOLES

Use a rubber mallet to carefully tap the bearings into the holes so that the bearing face is perfectly parallel to the pendulum face. You may need to slightly enlarge the hole with a file or rotary tool. The bearing should fit snugly in the hole, but don't damage it by pressing too hard or whacking it too roughly with the mallet.

If the fit is too tight, slightly enlarge the hole. If you inadvertently oversize a hole and the fit is too loose, use fast-setting epoxy to secure the bearing in place.

4. MAKE THE PIVOT ROD

Cut a 3" length of 8mm steel rod (or 5⁄16" wooden dowel). This pivot rod will connect the pendulum to the support stand.

If you're using steel rod, I recommend that you slightly flatten the end of the rod nearest the pendulum, so the pendulum won't fall off when it's pushed forcefully. Grip the steel rod near the end in a vise, and hammer the end carefully to flare it just slightly.

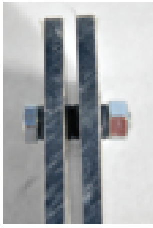
5. BUILD A SUPPORT FOR THE PENDULUM

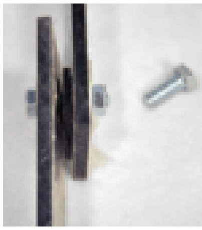
To support the pendulum, you can simply clamp a piece of wood to a vise on your workbench, or you can build a more elaborate stand from wood or metal. The only requirement is that the stand be solid and immovable when the pendulum is given a heavy push.

Attach the pendulum to the stand by drilling a 1½"-deep horizontal hole in the stand to accept the 3"-long pivot rod. To friction-fit the rod into the stand, this hole should be drilled with a 7mm or 5⁄16" bit. Also, the hole must be high enough so that no part of the pendulum touches the ground or benchtop when it swings. Insert the rod into the hole.

If you drilled entirely through the stand, I recommend putting an 8mm shaft collar on the opposite end of the shaft so it doesn't come out of the stand while spinning.

6. ASSEMBLE YOUR DOUBLE PENDULUM





To attach pendulum bearings to each other, use 5⁄16" bolts with nuts to make free pivots or fixed joints.

To adjust spacing between the pendulums, put nylon washers or other hard spacers on the bolt between the bearings to help space a pivoting joint. Place a flat rubber washer between the bearings to create a fixed, non-pivoting joint when the nut is tightened.

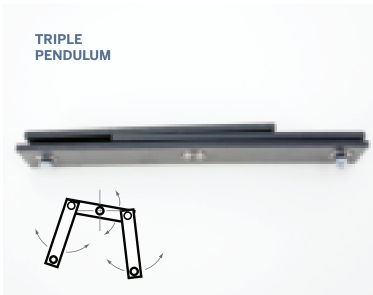
Build these 3 basic configurations following the diagrams:

Simple Double Pendulum
Mount one of the end bearings of the long pendulum onto the support rod or dowel attached to the stand. On the opposite end of the long pendulum, attach a short pendulum, using a pivoting joint.

Triple Pendulum
When all 3 pieces are allowed to swing from pivoting bearings, the system becomes a triple pendulum.

Don't push the bearing onto the pivot rod so far that the rod extends beyond the face of the bearing. If you do, it will interfere with the motion of the smaller pendulum.

You'll need to space the pendulums apart using plastic washers on the pivots so that they don't interfere with each other as they rotate.



Rott's Pendulum
Place the support pivot in the center bearing of the long pendulum. Then attach the 2 short pendulums to either end of the long one, fixing one of the joints at 90° and allowing the other joint to pivot freely. This creates a big L-shaped pendulum and a smaller side pendulum.

FINISH X **NOW GO USE IT »**

USE IT.

PUSH IT REAL GOOD

EXPERIMENTS TO TRY

Set up your double pendulum and give it a little push. The motion is not particularly interesting. Then give it a forceful push. The chaotic motion of the pendulum becomes fascinating!

When you give the triple pendulum a strong push, it exhibits a similar chaotic motion. But as the system settles down, the 2 end pendulums start to swing back and forth in unison; that is, the second pendulum swings in resonance with the first.

Some more experiments to try:

» Note when the smaller pendulum changes direction and try to predict what initial conditions (e.g. position at time of push, force of push) result in similar direction changes.

» Rearrange the pendulums, bearing support, and bearing types to vary motions and behaviors.

» Affix a 3-volt battery to the bottom of the pivoting pendulum. Tape the longer lead of an LED to the positive side of the battery and the shorter lead to the negative side; the LED will light. Take a time-lapse photograph of the LED as it moves in chaotic fashion. Try different pushes.

VARIATIONS TO CONSIDER

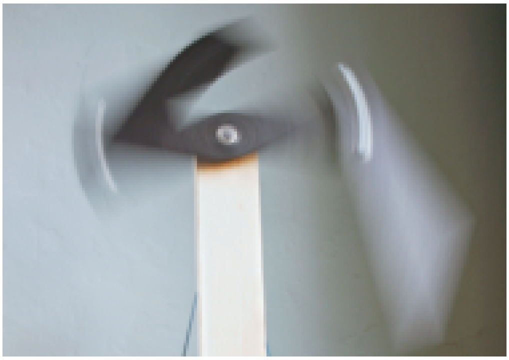
Our pendulum has been designed so that it's simple to choose different pendulum lengths, support bearing locations, and joint types. With different pendulum geometries, a great variety of motions, from regular to chaotic, may be obtained. You can make new parts to experiment further.

Pendulum length: Parts may be cut to any size. The length affects the behavior of the system.

Support location: The placement of the support bearing controls the motion of the pendulum. Move it slightly or greatly off-center to see what happens.

Joint type: Experiment with pivoting joints and fixed joints at various angles to see how these affect motion.

Photograph by Sam Murphy



ROTT'S PENDULUM: HAVE IT BOTH WAYS

Involving a number of daunting differential equations, the mathematics that describe what's going on in a Rott's pendulum are extremely complex. But the upshot is that at small amplitudes (when given a gentle push) the 2 pendulums will remain in motion for a far longer time than either pendulum would move alone.

If you crunch through the math you'll find that when the ratio of the length of the cross-member of the L-shaped pendulum to the length of the other 2 legs is 1.283567 to 1, the resonant frequencies of the 2 pendulums are integer ratios of one another, and the 2 parts, although much different in shape, are resonantly coupled.

While it's difficult to make a Rott's pendulum precisely enough to exhibit perfect resonant coupling, with enough care, you can demonstrate the phenomenon. Give a well-made Rott's pendulum a small shove and it swings on and on and on; give it a big push and it will exhibit the wild, chaotic behavior characteristic of any well-made, low-friction double pendulum. It's the best of both worlds.

See Bill Gurstelle's video of building a double pendulum: makezine.com/go/doublependulum